Bravo

$\frac{24}{25} = 96\%$

Basic Calculus /Math 130

Test 1

Name… ……………………… *GUST#.C.*

Time allowed 50 min. . Non scientific calculators are allowed.

1. Use the **definition of derivative (four step method)** to find $f'(x)$.

$$\frac{f(h+x)-f(x)}{h}$$

$$f(x) = \sqrt{x+2}$$

(4 p)

Step 1 $f(h+x) = \sqrt{(h+x)+2}$

Step 2 $f(h+x) - f(x) = \sqrt{(h+x)+2} - \sqrt{(x+2)}$

Step 3 $$\frac{f(h+x)-f(x)}{h} = \frac{\sqrt{(h+x)+2} - \sqrt{(x+2)}}{h} \cdot \frac{\sqrt{(h+x)+2} + \sqrt{(x+2)}}{\sqrt{(h+x)+2} + \sqrt{(x+2)}}$$

$$\sqrt{(h+x)+2} - \sqrt{(x+2)} \cdot \sqrt{(h+x)+2} + \sqrt{(x+2)}$$

$$A - B \quad A + B = A^2 - B^2$$

$$= \left(\sqrt{(h+x)+2}\right)^2 - \left(\sqrt{x+2}\right)^2$$

$$= (h+x)+2 - (x+2) = (h + \cancel{x} + \cancel{2}) - \cancel{x} - \cancel{2}$$

$$= \frac{1\,\cancel{h}}{\cancel{h}\left(\sqrt{(h+x)+2} + \sqrt{(x+2)}\right)} = \frac{1}{\sqrt{(h+x)+2} + \sqrt{(x+2)}}$$

Step 4 $$\lim_{h\to 0} \frac{f(h+x)-f(x)}{h} = \lim_{h\to 0} \frac{1}{\sqrt{x+2} + \sqrt{x+2}} = \boxed{\frac{1}{2\sqrt{x+2}}}$$

2. Find the limit

(9 p.)

a. $\lim_{x\to\infty} \frac{x^3-6}{2-x}$ $\frac{+}{-}$

$\lim_{x\to\infty} \frac{x^3}{-x} = \boxed{-\infty}$ ✓

3p.

b. $\lim_{x\to -3} \frac{x^2+6x+9}{x+3} = \frac{(x+3)(x+3)}{(x+3)}$ ✓

simplify

$\lim_{x\to -3} (x+3) = (-3)+3 = \boxed{0}$

3p.

c. $\lim_{x\to 1} \frac{4-x}{x-1}$ form $\frac{\#}{0} = \pm\infty$

$\lim_{x\to 1} \frac{4-x}{x-1} = \boxed{DNE}$ → maybe $+\infty$ or $-\infty$??

$\lim_{x\to 1} \frac{4-x}{x-1}$ 2p.

$\lim_{x\to 1^+} \frac{4-x}{x-1} = \frac{+}{+} = +\infty$

$\lim_{x\to 1^-} \frac{4-x}{x-1} = \frac{+}{-} = -\infty$

You should check!

$\therefore \lim_{x\to 1} \frac{4-x}{x-1} = DNE$

R= Price . X

P= R - Cost

3. Find the **marginal** profit function if the price and cost functions are given. derivative

$p(x)=25-0.05x$

$C(x)=100+0.2x$

$\boxed{P' = 0.1X + 24.8}$

marginal profit

$R = \text{Price} \cdot X$

$= (25 - 0.05X \cdot X)$

$= 25X - 0.05X^2$

(4 p.)

$P = R - \text{Cost}$

$= (25X - 0.05X^2) - (100 + 0.2X)$

$P = 25X - 0.05X^2 - 100 - 0.2X$

$= 0.05X^2 + 24.8X - 100$

4p.

4. Determine where the function is continuous. Express the answer in interval notation.

$$f(x)=\begin{cases}\sqrt[3]{x} & \text{if } x\le 1 \\ x^2-x+1 & \text{if } x>1\end{cases}$$

- #s $(-\infty, 1)$

+ #s $(1, \infty)$

(4 p.)

4p.

$\lim_{x \to 1^-} \sqrt[3]{X} = \sqrt[3]{1} = 1$

$\lim_{x \to 1^+} X^2 - X + 1 = (1)^2 - 1 + 1 = 1$

$\lim_{X \to 1} f(X) = 1$

$f(1) = \sqrt[3]{1} = 1$

$\therefore$ function is Continuous at $x=1$

$(-\infty, \cancel{1) \cup (1}, +\infty)$

$(-\infty, \infty)$

$\sqrt[n]{x^m} = x^{m/n}$

$\sqrt[2]{x} = x^{1/2}$

5. Find the derivative.

a. $f(x) = 5 - 2x^{\frac{3}{2}} - \sqrt{x} + \frac{1}{x^4}$

(4 p.)

$f(x) = 5 - 2x^{3/2} - x^{\frac{1}{2}} + x^{-4}$

$f'(x) = \frac{3}{2} \cdot (-2) x^{3/2 - 1} - \frac{1}{2} \cdot x^{1/2 - 1} + (-4) \cdot x^{-4-1}$

$\boxed{f' = -3x^{1/2} - 0.5x^{-1/2} - 4x^{-5}}$

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