

$$\frac{26.5}{30} = 88\%$$

Math 130
Midterm Exam

GUST# _____ .Name.. _____

Time allowed 50 min. Only nonscientific calculators are allowed.

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{1}{x^2} = x^{-2}$$

$$\ln x^2 = \frac{1}{x^2} \cdot 2x \quad (4 \text{ p})$$

$$\ln x^2$$

1. Find the derivative (do not simplify).

$$\ln x^2 = 2 \ln x \Rightarrow \text{The deriv is } 2 \cdot \frac{1}{x} = \boxed{\frac{2}{x}}$$

a) $f(x) = 3^x + \ln x^2 - \sqrt[3]{x^2} + \frac{4}{x^5}$

$$f(x) = 3^x + \ln x^2 - x^{\frac{2}{3}} + 4x^{-5}$$

OR $\ln x^2$

$$\frac{1}{x^2} \cdot (2x) = \frac{2x}{x^2} = \boxed{\frac{2}{x}}$$

check

$$f' = 3^x \ln 3 + 2 \ln \frac{1}{x} - \frac{2}{3} x^{-\frac{1}{3}} - 20x^{-6}$$

$$\boxed{3 \cdot 5}$$

a) $y = \sqrt[4]{(x^3 - 7)^3}$

$$y = (x^3 - 7)^{\frac{3}{4}}$$

$$y' = \frac{3}{4} (x^3 - 7)^{-\frac{1}{4}} \cdot (3x^2)$$

(3 p)

$$\boxed{3}$$

T	B
$x^2 + 2$	$x - 3$
$2x$	1

b) $y = \frac{x^2 + 2}{x - 3}$

Important (i)

$$y' = \frac{2x(x-3) + (x^2+2)}{(x-3)^2} = \frac{2x^2 - 6x + x^2 + 2}{(x-3)^2} \quad (3 \text{ p})$$

$$\boxed{2 \cdot 5}$$

c) $y = e^{(x^2 + 7x)}$

$$y' = e^{(x^2 + 7x)} \cdot (2x + 7)$$

$$\boxed{2}$$

(2 p)

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2. Find the limit (if it exists). If needed use ∞ or $-\infty$.

a) $\lim_{x \rightarrow 2^+} \frac{2x}{x^2+5}$

(6 p)

$$\lim_{x \rightarrow 2^+} \frac{2x}{x^2+5} = \frac{2 \cdot (2)}{(2)^2+5} = \frac{4}{9} \quad \checkmark \quad (2 \text{ p.})$$

b) $\lim_{x \rightarrow \infty} \frac{2x-1}{5+x^3}$

Power Top is less than Bottom

(4 = 0)

$$\lim_{x \rightarrow \infty} \frac{2x}{x^3} = \lim_{x \rightarrow \infty} \frac{2}{x^2} = 0$$

(1.5 p)

↑ Important (i)

c) $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1}$

Form $\frac{0}{0}$ Factor ... $\frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$

Form $\frac{\infty}{0}$

$$\lim_{x \rightarrow -1} \frac{x+1}{x^2-1}$$

$$\lim_{x \rightarrow -1} \frac{x+1}{x^2-1} = -\infty$$

$$\lim_{x \rightarrow -1} \frac{x+1}{x^2-1} = +\infty$$

$$\therefore \lim_{x \rightarrow -1} \frac{x+1}{x^2-1} = \text{DNE}$$

(Op.)

$$\lim_{x \rightarrow -1} \frac{1}{x-1} = \frac{1}{-2}$$

3.5

3. Given function $f(x) = (x^2 + 1)(2 - x)$. Find the value of x at which the slope of the tangent line to function is zero. $m=0$ $y-y_1 = m(x-x_1)$

$$f(x) = (x^2 + 1)(2 - x) = 0$$

$x^2 + 1$	$2 - x$
$2x$	-1

$x = -1$ $x = -2$
 $x = +1$

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Op.

(5 p)

$f' = 0$

$$f' = 2x(2-x) + (-1 \cdot x^2 + 1)$$

$$f' = 4x - 2x^2 + (-x^2 - 1)$$

$$4x - 2x^2 - x^2 - 1$$

$$4x - 3x^2 - 1$$

make it = 0

$4x - 3x^2 - 1 = 0$

$-3x^2 + 4x - 1 = 0$ 4. Find the marginal profit function if the price and cost functions are given.

$p = 1296 - 0.12x^2$; $0 \leq x \leq 80$
 $C(x) = 830 + 396x$

$P = R - C$ $R = \text{Price} \cdot X$

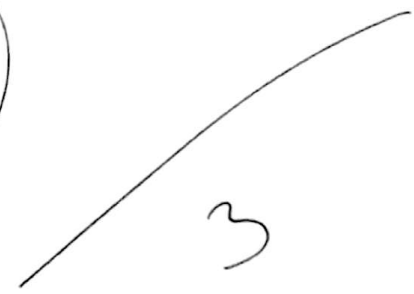
$R = P \cdot x$
 $(1296 - 0.12x^2) \cdot (x) = 1296x - 0.12x^3$

(3 p)

$P = R - C$
 $1296x - 0.12x^3 - (830 + 396x)$
 $= 1296x - 0.12x^3 - 830 - 396x$
 $= -0.12x^3 + 900x - 830$

$P' = -3 \cdot 0.12x^2 + 900$

3p.



5. Find the time required for the money in your account to triple, if the rate of interest is 8% compounded continuously. $A = P \cdot e^{rt}$

Given: $r = 0.08$

$$A = P \cdot e^{r \cdot t}$$

$$3P = P \cdot e^{r \cdot t}$$

$$3 = e^{r \cdot t} \quad \text{take ln for both sides.}$$

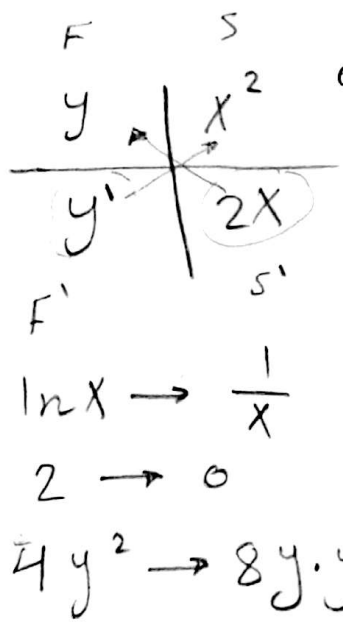
$$\ln 3 = \ln e^{0.08 \cdot t}$$

$$\ln 3 = 0.08t \ln e = 1$$

$$\frac{\ln 3}{0.08} = 0.08t \rightarrow t = \frac{\ln 3}{0.08} = 13.7 \approx 14 \text{ years to triple the money.}$$

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6. Use implicit differentiation to find the derivative of $y(x)$ given $\ln x + 2 - 4y^2 = yx^2$



$$\frac{1}{x} - 8y \cdot y' = y' \cdot x^2 + 2x \cdot y$$

$$\frac{1}{x} - 8y \cdot y' - y' \cdot x^2 - 2x \cdot y = 0 \quad (5 \text{ p})$$

$$y' (-8y - x^2) = -\frac{1}{x} + 2x \cdot y$$

$$y' = \frac{2x \cdot y - \frac{1}{x}}{-8y - x^2}$$

5

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