$\frac{26.5}{30} = 88\%$

Math 130
Midterm Exam

GUST# .Name..

Time allowed 50 min. Only nonscientific calculators are allowed.

$\ln x^2$

$\sqrt{x} = x^{\frac{1}{2}}$

1. Find the derivative (**do not** simplify).

$\ln x^2 = 2\ln x \Rightarrow$ The deriv is $2 \cdot \frac{1}{x} = \boxed{\frac{2}{x}}$

$\frac{1}{x^2} = x^{-2}$

OR

$\ln x^2 = \frac{1}{x^2} \cdot (2x) = \frac{2x}{x^2} = \boxed{\frac{2}{x}}$ (chain)

$\ln x^2 = 2x$

a) $f(x) = 3^x + \ln x^2 - \sqrt[3]{x^2} + \frac{4}{x^5}$ (4 p)

$f(x) = 3^x + \ln x^2 - x^{\frac{2}{3}} + 4x^{-5}$

$f' = 3^x \cdot \ln 3 + 2\ln\frac{1}{x} - \frac{2}{3}x^{-\frac{1}{3}} - 20x^{-6}$ ✓ ? ✓ ✓

3.5

a) $y = \sqrt[4]{(x^3-7)^3}$ (3 p)

$y = (x^3-7)^{\frac{3}{4}}$

$y' = \frac{3}{4}(x^3-7)^{-\frac{1}{4}} \cdot (3x^2)$

3

T	B
x^2+2	$x-3$
$2x$	1

T' B'

b) $y = \frac{x^2+2}{x-3}$ (3 p)

Important!

$y' = \frac{2x(x-3) - (x^2+2)}{(x-3)^2} = \frac{2x^2-6x-x^2-2}{(x-3)^2}$

2.5

c) $y = e^{(x^2+7x)}$ (2 p)

$y' = e^{(x^2+7x)} \cdot (2x+7)$ ✓

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2. Find the limit (if it exists). If needed use ∞ or -∞.

a) $\lim_{x\to 2^+} \frac{2x}{x^2+5}$ (6 p)

$$\lim_{x\to 2^+} \frac{2x}{x^2+5} = \frac{2\cdot(2)}{(2)^2+5} = \frac{4}{9}$$ ✓ 2p.

b) $\lim_{x\to\infty} \frac{2x-1}{5+x^3}$ $\frac{1}{x}$ $\frac{2}{x^2}$

Power Top is less than Bottom $\lim = 0$

$$\lim_{x\to\infty} \frac{2x}{x^3} = \lim_{x\to\infty} \frac{2}{x^2} = 0$$ 1.5 p

Important :)

c) $\lim_{x\to -1} \frac{x+1}{x^2-1}$ For $\frac{0}{0}$ Factor ... $\frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$

For num $\neq 0$

$$\lim_{x\to -1^+} \frac{x+1}{x^2-1} = -\infty$$

$$\lim_{x\to -1^-} \frac{x+1}{x^2-1} = +\infty$$

$\therefore \lim_{x\to -1} \frac{x+1}{x^2-1} = \text{DNE}$

0 p.

$$\lim_{x\to -1} \frac{1}{x-1} = \boxed{\frac{1}{-2}}$$

3.5

3. Given function $f(x) = (x^2 + 1)(2 - x)$. Find the value of x at which the slope of the tangent line to function is zero. (5 p)

$m = 0$ $\quad y - y_1 = m(x - x_1)$, $m = 0$

$f(x) = (x^2+1)(2-x) = 0$

F: $x^2 + 1$ $\quad$ S: $2 - x$

$x = -1$ $\quad x = -2$

$x = +1$

$f' = 2x(2-x) + (-1 \cdot x^2 + 1)$

$f' = 4x - 2x^2 + (-x^2 - 1)$

$4x - 2x^2 - x^2 - 1$

$4x - 3x^2 - 1$

make it $= 0$

$4x - 3x^2 - 1 = 0$

$-3x^2 + 4x - 1 = 0$

4. Find the marginal profit function if the price and cost functions are given. (3 p)

$p = 1296 - 0.12x^2 \quad ; 0 \le x \le 80$

$C(x) = 830 + 396x$

$P = R - C$ $\quad R = \text{Price} \cdot x$

$R = P \cdot x$

$(1296 - 0.12x^2) \cdot (x) = 1296x - 0.12x^3$

$P = R - C$

$1296x - 0.12x^3 - (830 + 396x)$

$= 1296x - 0.12x^3 - 830 - 396x$

$= -0.12x^3 + 900x - 830$

$P' = -3 \cdot 0.12x^2 - 900$

5. Find the time required for the money in your account to triple, if the rate of interest is 8% compounded continuously. (4 p)

Given: $r = 0.08$

$A = P \cdot e^{r \cdot t}$

$3P = P \cdot e^{r \cdot t}$

$3 = e^{r \cdot t}$ take ln for both sides..

$\ln 3 = \ln e^{0.08 \cdot t}$

$\ln 3 = 0.08\, t \ln e$ ($\ln e = 1$)

$\frac{\ln 3}{0.08} = \frac{0.08\, t}{0.08} \longrightarrow t = \frac{\ln 3}{0.08} = 13.7 \cong 14$ years to triple the money.

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6. Use implicit differentiation to find the derivative of $y(x)$ given $\ln x + 2 - 4y^2 = yx^2$ (5 p)

F S

$y \cdot x^2$

$y' \cdot 2x$

F' S'

$\ln x \rightarrow \frac{1}{x}$

$2 \rightarrow 0$

$4y^2 \rightarrow 8y \cdot y'$

$\frac{1}{x} - 8y \cdot y' = y' \cdot x^2 + 2x \cdot y$

$\frac{1}{x} - 8y \cdot y' - y' \cdot x^2 - 2x \cdot y = 0$

$y'(-8y - x^2) = -\frac{1}{x} + 2x \cdot y$

$\boxed{y' = \frac{2x \cdot y - \frac{1}{x}}{-8y - x^2}}$

5

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